

ERRATUM TO "HOMOTOPY IN FUNCTOR CATEGORIES"

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Lemma 3.1 is incorrect as stated as, of course, is the argument there advanced for it. It should be amended as follows.

LEMMA 3.1. *If $X \rightarrow \tilde{X}$ is a free prolongation as above then it is a cofibration in $\mathfrak{T}^{\mathbf{C}}$. If $\mathbf{C}' \rightarrow \mathbf{C} \circ (J^{\text{op}} \times J)$ is a cofibration in $\mathfrak{T}^{\mathbf{C}'^{\text{op}} \times \mathbf{C}'}$ then $(J^*\theta)\eta_A: A \rightarrow J^*\tilde{X}$ is a cofibration in $\mathfrak{T}^{\mathbf{C}'}$.*

Proposition 4.4 (which is independent of §3) implies that the upper square of the diagram

$$\begin{array}{ccc} \mathbf{C}' \otimes_{\mathbf{C}} J^*X = J^*X & \rightarrow & A \\ \downarrow \eta_{J^*X} & & \downarrow \eta_A \\ J^*\text{Lan}_J J^*X & \rightarrow & J^*\text{Lan}_J A = \mathbf{C} \circ (J^{\text{op}} \times J) \otimes_{\mathbf{C}} A \\ \downarrow J^*\varepsilon_X & & \downarrow J^*\theta \\ J^*X & \rightarrow & J^*\tilde{X} \end{array}$$

is a 2-cofibration. Since J^* is a left adjoint, the lower square is a pushout. Since $(J^*\varepsilon_X)\eta_{J^*X} = 1$, the conclusion follows in routine fashion.

The additional hypothesis asks to be characterized as the assertion that $\mathbf{C}' \subset \mathbf{C}$ is a *cofibered subcategory*. There have been other uses of the adjective in category theory, but they should lead to no confusion in this context. The rest of §3, including especially Theorem 3.5, evidently requires this stronger hypothesis as well, so that the results below on homotopy relative to a subcategory are to be asserted only when the subcategory is cofibered.

This restriction is real, but perhaps not unduly onerous. Its scope is indicated by the following two examples: (i) In general, $\mathbf{C}_0 \subset \mathbf{C}$ is cofibered if and only if, for each object c , $1_c \in \mathbf{C}(c, c)$ is cofibered in \mathfrak{T} . (ii) If \mathbf{C} is \mathfrak{T} -discrete then $\mathbf{C}' \subset \mathbf{C}$ is cofibered if and only if, for any composition xyz in \mathbf{C} with x, z and xyz in \mathbf{C}' , y must also be in \mathbf{C}' .

Thus, for example, if \mathbf{C} is the free group on one generator t and \mathbf{C}' the submonoid generated by t then \mathbf{C}' is not cofibered in \mathbf{C} . Taking $X = \emptyset$, $A = \{x, y\}$, $tx = ty = x$, we get a counterexample to the original (false) statement of Lemma 3.1.

Alex Heller, *Homotopy in functor categories*, Trans. Amer. Math. Soc. **272** (1982), 185–202.

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